

Canonical Flow in the Space of Gauge Parameters

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Abstract

Gauge dependence of one-particle irreducible (1-PI) amplitudes in $SU(N)$ Yang-Mills theory is shown to be generated by a canonical flow with respect to (w.r.t.) the extended Slavnov-Taylor (ST) identity, induced by the transformation of the gauge parameter α under the BRST symmetry. For linear covariant gauges, the analytic expansion in α of 1-PI amplitudes is given in terms of coefficients evaluated in the Landau gauge and of derivatives w.r.t. α of the generating functional of the flow. An application to the gauge flow of the gluon propagator is considered.

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1 Introduction

While physical quantities have to be gauge-invariant, it sometimes happens that particular gauges are computationally more suited than others in the study of several properties of gauge theories.

QCD provides a number of examples of this phenomenon. Just to mention a few, the computation of the effective action for the Color Glass Condensate and of the ensuing evolution equations is most easily carried out in the Light-Cone gauge for the semi-fast gluons [1]-[3], so that gauge-invariance is not manifest from the beginning.

However it has been recently proven [4] that gauge-invariance of the evolution equations indeed holds as a consequence of a suitable Slavnov-Taylor (ST) identity, arising from the BRST symmetry of QCD in the presence of the classical fast gluon backgrounds, so that any gauge choice for the semi-fast modes can in fact be adopted.

A perhaps more striking example is the existence of massive solutions of appropriate truncations to the QCD Schwinger-Dyson equations [5, 6, 7], that has been established in the Landau gauge, confirming lattice simulations again carried out in the Landau gauge, both in SU(2) [8] and in SU(3) [9].

Moreover, the study of the Kugo-Ojima function is also usually formulated in the Landau gauge [10].

In the study of the IR properties of QCD it is therefore particularly important to try to establish a method, as general as possible, in order to ease the comparison of computations carried out in different gauges.

On the formal side, it has been known since a long time that gauge dependence of amplitudes can be studied algebraically through (generalized) Nielsen identities [11, 12, 13]. Formally these identities can be derived by extending the action of the BRST differential s to the gauge parameters. Their BRST variation is given by classical anticommuting variables paired into a so-called BRST doublet (i.e. a pair of variables u, v such that $su = v, sv = 0$).

This technical device bears a close analogy with the algebraic treatment of gauge theories in the presence of a classical background connection [14]-[17], where again the classical background field \hat{A}_μ is paired under the BRST differential s with a classical anticommuting source Ω_μ .

We remark that this mathematical structure indeed arises very naturally if one imposes both the BRST and the antiBRST symmetry of the underlying gauge theory [18]. This requirement allows to obtain a local antighost equation, valid in any Lorentz-covariant gauge, by extending the local antighost equation originally derived in the Landau gauge [19].

The resulting extended ST identity turns out to completely determine the dependence on the background \hat{A}_μ of the vertex functional Γ through a canonical transformation w.r.t. the Batalin-Vilkovisky bracket of the theory,

induced by the generating functional $\Psi_\mu \equiv \frac{\delta\Gamma}{\delta\Omega_\mu}$ [20, 21].

Moreover it has been shown that the solution to the extended ST identity can be written in terms of a certain Lie series, while naive exponentiation would fail due to the dependence of Ψ_μ on the background [22].

In the present paper we will extend these results to the canonical flow induced by the extended ST identity in the space of gauge parameters. We will show that such a flow can be derived on the basis of the ST identity only (so that one can dispose of the equations ensuring the stability of the gauge-fixing for some particular gauge choices, like the Nakanishi-Lautrup and the ghost equation [23] in Lorentz-covariant gauges).

Then we will obtain the explicit form of the Lie series which gives the expansion of the effective action in powers of the gauge parameter α (by assuming analyticity in α). The coefficients are given by amplitudes evaluated in the theory at $\alpha = 0$ (e.g., in the example of Lorentz-covariant gauges, in terms of Landau gauge amplitudes) plus some contributions induced by the α -dependence of the generating functional of the canonical flow.

We will then discuss in some detail the gauge flow relating the gluon propagator in the Landau and in the Lorentz-covariant gauge. The relations derived here are valid in perturbation theory. It might be tempting to speculate whether they can also be applied in the non-perturbative regime. This problem is however well beyond the scope of the present paper, since it involves the need of a deeper discussion of the analyticity of the gluon propagator in the gauge parameter around the Landau gauge point at $\alpha = 0$, which might be spoiled beyond perturbation theory.

The paper is organized as follows. In Sect. 2 we introduce our notations and derive the extended ST identity for SU(N) Yang-Mills theory. In Sect. 3 we construct the canonical flow governing the gauge dependence of the amplitudes and discuss the role of the gauge dependence of the generating functional. In Sect. 4 the connection between the Lorentz-covariant gauges and the Landau gauge is analyzed. Finally in Sect. 5 we discuss an application of the formalism to the gluon propagator. Conclusions are given in Sect. 6.

2 Classical Action

Let us consider pure SU(N) Yang-Mills theory with classical action

$$S = -\frac{1}{4g^2} \int d^4x G_{a\mu\nu}^2, \quad (1)$$

with the field strength given by

$$G_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + f_{abc} A_{b\mu} A_{c\nu} \quad (2)$$

and f_{abc} the $SU(N)$ structure constants. The inclusion of matter fields does not introduce further complications in the ensuing analysis.

The usual quantization procedure based on the BRST symmetry requires the introduction in the tree-level vertex functional of a gauge-fixing function \mathcal{F}_a through the coupling with the Nakanishi-Lautrup multiplier field b_a [24]:

$$S_{g.f.} = - \int d^4x b_a \mathcal{F}_a \quad (3)$$

(the minus sign is inserted for notational convenience). For the present purposes we do not need to specify the exact form of the gauge-fixing function \mathcal{F}_a . The only condition is that it should allow the inversion of the tree-level 2-point functions in the $A_{a\mu} - b_b$ sector, yielding the tree-level propagators for the gauge and Nakanishi-Lautrup multiplier fields. \mathcal{F}_a might also depend on some parameters λ_i . For instance, one might choose

$$\mathcal{F}_a = (1 - \lambda) \partial^\mu A_{a\mu} + \lambda \partial^i A_i \quad (4)$$

interpolating between the Lorentz-covariant gauge ($\lambda = 0$) and the Coulomb gauge ($\lambda = 1$). Another example is the Slavnov-Frolov regularization of the Light Cone gauge [25]

$$\mathcal{F}_a = A_- + \lambda \partial_- A_- \quad (5)$$

where $A_- = A_0 - A_3$ and $\partial_- = \partial_0 - \partial_3$. Green functions are evaluated at $\lambda \neq 0$ and then one takes the limit $\lambda \rightarrow 0$.

Gauge invariance lost after the gauge-fixing procedure is promoted to full BRST symmetry by adding to the classical action both the gauge-fixing and the ghost-dependent terms

$$\begin{aligned} S_{g.f.+gh} &= s \int d^4x \bar{c}_a \left(\frac{\alpha}{2} b_a - \mathcal{F}_a \right) \\ &= \int d^4x \left(\frac{\alpha}{2} b_a^2 - b_a \mathcal{F}_a + \bar{c}_a s \mathcal{F}_a \right), \end{aligned} \quad (6)$$

where the nilpotent BRST differential s acts as follows. On the gauge field it equals the gauge transformation, upon replacement of the gauge parameters with the ghost fields c_a

$$s A_{a\mu} \equiv D_\mu c_a = \partial_\mu c_a + f_{abc} A_{b\mu} c_c, \quad (7)$$

where $D_\mu c_a$ is the covariant derivative of the ghost field. The transformation of the ghost is in turn dictated by the nilpotency of s , i.e.

$$s c_a = -\frac{1}{2} f_{abc} c_b c_c. \quad (8)$$

The antighost \bar{c}_a and the Nakanishi-Lautrup multiplier field b_a form a BRST doublet [26, 27], i.e.

$$s\bar{c}_a = b_a, \quad sb_a = 0. \quad (9)$$

The parameter α reduces to the usual gauge parameter for Lorentz-covariant gauges when $\mathcal{F}_a = \partial A_a$.

2.1 Slavnov-Taylor Identity

Since the BRST variations of the gauge and ghost fields in Eqs.(7) and (8) are non-linear in the quantum fields, their renormalization requires the introduction of external sources known as antifields [28]. They are coupled to the BRST variation of the corresponding fields as follows

$$S_{a.f.} = \int d^4x \left(A_{a\mu}^* sA_{a\mu} - c_a^* sc_a \right). \quad (10)$$

The minus sign in front of $c_a^* sc_a$ is introduced for consistency with the Batalin-Vilkovisky (BV) bracket conventions of [28].

Then the tree-level vertex functional

$$\Gamma^{(0)} = S + S_{g.f.+gh} + S_{a.f.} \quad (11)$$

obeys the ST identity [29, 30]

$$\mathcal{S}(\Gamma^{(0)}) \equiv \int d^4x \left(\frac{\delta\Gamma^{(0)}}{\delta A_{a\mu}^*} \frac{\delta\Gamma^{(0)}}{\delta A_{a\mu}} - \frac{\delta\Gamma^{(0)}}{\delta c_a^*} \frac{\delta\Gamma^{(0)}}{\delta c_a} + b_a \frac{\delta\Gamma^{(0)}}{\delta \bar{c}_a} \right) = 0. \quad (12)$$

Notice that the linearity of the BRST transformation of the antighost \bar{c}_a does not strictly require the introduction of an antifield for \bar{c}_a .

The ST identity in Eq.(12) holds irrespectively of the particular form of the gauge-fixing function \mathcal{F}_a chosen. For some specific choices of the latter (e.g. linear covariant gauges or the Landau gauge) further identities arise, like the equation for the b -field and the ghost equation [23]. However, we will not rely on these identities in the following discussion.

2.2 BRST Variation of the Gauge Parameters

It has been known since a long time [12] that one can extend the BRST symmetry to act on the gauge parameters in such a way to derive an extended ST identity, leading to the so-called Nielsen identities [11, 12]. I.e. one defines

$$s\lambda_i = \theta_i, \quad s\theta_i = 0, \quad s\alpha = \theta \quad s\theta = 0. \quad (13)$$

Under the extended BRST symmetry, $S_{g.f.+gh}$ in Eq.(6) receives an additional contribution

$$\begin{aligned} S_{g.f.+gh} &= s \int d^4x \bar{c}_a \left(\frac{\alpha}{2} b_a - \mathcal{F}_a \right) \\ &= \int d^4x \left(\frac{\alpha}{2} b_a^2 - b_a \mathcal{F}_a + \bar{c}_a s \mathcal{F}_a + \frac{\theta}{2} \bar{c}_a b_a \right) \\ &\quad + \int d^4x \bar{c}_a \left(\frac{\partial \mathcal{F}_a}{\partial \lambda_i} \theta_i + \frac{\partial \mathcal{F}_a}{\partial \alpha} \theta \right) \end{aligned} \quad (14)$$

and the tree-level classical action fulfills the extended ST identity

$$\tilde{\mathcal{S}}(\Gamma^{(0)}) = \sum_i \theta_i \frac{\partial \Gamma^{(0)}}{\partial \lambda_i} + \theta \frac{\partial \Gamma^{(0)}}{\partial \alpha} + \mathcal{S}(\Gamma^{(0)}) = 0. \quad (15)$$

For non-anomalous theories this equation holds for the full vertex functional Γ :

$$\tilde{\mathcal{S}}(\Gamma) = \sum_i \theta_i \frac{\partial \Gamma}{\partial \lambda_i} + \theta \frac{\partial \Gamma}{\partial \alpha} + \mathcal{S}(\Gamma) = 0. \quad (16)$$

By taking a derivative w.r.t. θ and then setting θ, θ_i equal to zero one obtains the following Nielsen identity

$$\begin{aligned} \frac{\partial \Gamma}{\partial \alpha} \Big|_{\theta=\theta_i=0} &= - \int d^4x \left(\frac{\delta^2 \Gamma}{\partial \theta \delta A_{a\mu}^*} \frac{\delta \Gamma}{\delta A_{a\mu}} - \frac{\delta \Gamma}{\delta A_{a\mu}^*} \frac{\delta^2 \Gamma}{\partial \theta \delta A_{a\mu}} \right. \\ &\quad \left. - \frac{\delta^2 \Gamma}{\partial \theta \delta c_a^*} \frac{\delta \Gamma}{\delta c_a} - \frac{\delta \Gamma}{\delta c_a^*} \frac{\delta^2 \Gamma}{\partial \theta \delta c_a} + b_a \frac{\delta^2 \Gamma}{\partial \theta \delta \bar{c}_a} \right) \Big|_{\theta=\theta_i=0}. \end{aligned} \quad (17)$$

A similar equation holds for the derivative of Γ w.r.t. λ_i , once one takes a derivative of the extended ST identity in eq.(16) w.r.t. θ_i .

3 Canonical Flow for Gauge Parameters

There is a close formal analogy between Eq.(16) and the extended ST identity controlling the dependence on a background field configuration \hat{A}_μ [14]-[17]. This analogy relies on the fact that both the gauge parameters and the background configurations form BRST doublets (i.e. a couple of variables u, v , such that $su = v, sv = 0$) together with their BRST partners.

Assuming analyticity in the background field configuration, the solution to the extended ST identity can be obtained by a suitable Lie series [22] that allows to express all the coefficients in the expansion in powers of \hat{A}_μ in terms of Green functions evaluated at zero background. The failure of naive exponentiation and the need to use a Lie series arises from the dependence of the generating functional, controlling the background dependence, on the background itself.

The same technique can be used to obtain a Lie series for the expansion of the vertex functional in powers of α (or λ_i) in terms of amplitudes evaluated at $\alpha = 0$ (or $\lambda_i = 0$). Again one assumes analyticity in the parameter α (or λ_i) one is considering.

For that purpose it is convenient to rewrite the extended ST identity within the BV formalism [28]. Hence one introduces an antifield \bar{c}_a^* for the antighost \bar{c}_a as well as the antifield b_a^* for the Nakanishi-Lautrup field b_a . \bar{c}_a^* is coupled to b_a in the classical action, while b_a^* does not enter into $\Gamma^{(0)}$ (since $sb_a = 0$).

Then one defines the BV bracket as follows (we use only left derivatives)

$$\{X, Y\} = \int d^4x \sum_{\phi} \left[(-1)^{\epsilon_{\phi}(\epsilon_X+1)} \frac{\delta X}{\delta \phi} \frac{\delta Y}{\delta \phi^*} - (-1)^{\epsilon_{\phi^*}(\epsilon_X+1)} \frac{\delta X}{\delta \phi^*} \frac{\delta Y}{\delta \phi} \right]. \quad (18)$$

The sum runs over the fields $\phi = (A_{a\mu}, c_a, \bar{c}_a, b_a)$ and the corresponding antifields $\phi^* = (A_{a\mu}^*, c_a^*, \bar{c}_a^*, b_a^*)$. $\epsilon_{\phi}, \epsilon_{\phi^*}$ are the statistics of the field ϕ and the antifield ϕ^* . ϵ_X is the statistics of the functional X .

Then the extended ST identity (16) can be written as

$$\tilde{\mathcal{S}}(\Gamma) = \sum_i \theta_i \frac{\partial \Gamma}{\partial \lambda_i} + \theta \frac{\partial \Gamma}{\partial \alpha} + \frac{1}{2} \{\Gamma, \Gamma\} = 0. \quad (19)$$

By taking a derivative w.r.t. θ (but the argument goes in the same way if one takes a derivative w.r.t. θ_i) one finds

$$\left. \frac{\partial \Gamma}{\partial \alpha} \right|_{\theta=\theta_i=0} = - \left\{ \left. \frac{\partial \Gamma}{\partial \theta} \right|_{\theta=\theta_i=0}, \Gamma \right\} \Big|_{\theta=\theta_i=0}. \quad (20)$$

This equation shows that the derivative of the vertex functional w.r.t. α is obtained by a canonical transformation (w.r.t. the BV bracket) induced by the generating functional $\Psi \equiv \frac{\partial \Gamma}{\partial \theta}$. Since the latter in general depends on α , one cannot solve Eq.(20) by simple exponentiation and one needs to make recourse to a Lie series.

For that purpose, one introduces the operator

$$\Delta_{\Psi} = \{\cdot, \Psi\} + \frac{\partial}{\partial \alpha}. \quad (21)$$

Then the vertex functional Γ is given by the following Lie series [22]

$$\Gamma = \sum_{n \geq 0} \frac{1}{n!} \alpha^n [\Delta_{\Psi}^n \Gamma_0]_{\alpha=0} \quad (22)$$

where Γ_0 is the vertex functional at $\alpha = 0$. Notice that one must afterwards take the limit $\alpha \rightarrow 0$ (although the operator Δ_{Ψ} is applied on the functional Γ_0 , which is α -independent) since a residual α -dependence may arise (and

in general indeed arises) from the differentiation w.r.t. α of the generating functional Ψ .

We also remark that Eq.(22) holds irrespectively of the form of the gauge-fixing function \mathcal{F}_a (and in particular is independent of the existence of a b -equation and of a ghost equation, guaranteeing the stability of the gauge-fixing in certain classes of gauge [23]).

4 Lorentz-covariant Gauges

Let us illustrate the above formalism in the simple example of the Lorentz-covariant gauge, i.e. let us choose

$$\mathcal{F}_a = \partial A_a. \quad (23)$$

Then one gets

$$S_{g.f.+gh} = \int d^4x \left(\frac{\alpha}{2} b_a^2 - b_a \partial A_a + \bar{c}_a \partial^\mu (D_\mu c)_a + \frac{\theta}{2} \bar{c}_a b_a \right). \quad (24)$$

The propagators are

$$\begin{aligned} \Delta_{A_\mu^a A_\nu^b} &= -i \delta^{ab} \left(\frac{1}{p^2} T^{\mu\nu} + \frac{\alpha}{p^2} L^{\mu\nu} \right), & \Delta_{b_a A_{b\mu}} &= -\delta^{ab} \frac{p^\mu}{p^2}, \\ \Delta_{b_a b_b} &= 0, & \Delta_{c_a \bar{c}_b} &= \delta_{ab} \frac{i}{p^2}. \end{aligned} \quad (25)$$

$T^{\mu\nu} = g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}$ is the transverse projector, $L^{\mu\nu} = \frac{p^\mu p^\nu}{p^2}$ is the longitudinal one.

For Lorentz covariant gauges the b -equation and the ghost equation hold:

$$\begin{aligned} \frac{\delta \Gamma}{\delta b_a} &= \alpha b_a - \partial A_a, \\ \frac{\delta \Gamma}{\delta \bar{c}_a} &= \partial^\mu \frac{\delta \Gamma}{\delta A_{a\mu}^*} - \frac{\theta}{2} b_a. \end{aligned} \quad (26)$$

The first of the above equations implies that the b -dependence is confined at tree level. The second equation in turn implies that at higher orders $n \geq 1$ Γ can depend on \bar{c}_a only through the combination

$$\tilde{A}_{a\mu}^* = A_{a\mu}^* - \partial_\mu \bar{c}_a. \quad (27)$$

By redefining the antifield $A_{a\mu}^*$ according to the above equation and by introducing the reduced functional $\tilde{\Gamma} = \Gamma - \int d^4x \frac{\alpha}{2} b_a^2 + \int d^4x b_a \partial A_a$, the BV bracket can be restricted to the variables $(A_{a\mu}, \tilde{A}_{a\mu}^*)$ and (c_a, c_a^*) and the flow equation reads

$$\left. \frac{\partial \tilde{\Gamma}}{\partial \alpha} \right|_{\theta=\theta_i=0} = - \int d^4x \left[\frac{\delta \Psi}{\delta A_{a\mu}} \frac{\delta \tilde{\Gamma}}{\delta \tilde{A}_{a\mu}^*} + \frac{\delta \Psi}{\delta \tilde{A}_{a\mu}^*} \frac{\delta \tilde{\Gamma}}{\delta A_{a\mu}} - \frac{\delta \Psi}{\delta c_a} \frac{\delta \tilde{\Gamma}}{\delta c_a^*} - \frac{\delta \Psi}{\delta c_a^*} \frac{\delta \tilde{\Gamma}}{\delta c_a} \right]. \quad (28)$$

Since we will only use $\tilde{\Gamma}$ in what follows, we will simply write Γ for $\tilde{\Gamma}$. The Lie operator Δ_Ψ is

$$\begin{aligned}\Delta_\Psi(X) &= \{X, \Psi\} + \frac{\partial X}{\partial \alpha} \\ &= \int d^4x \left[\frac{\delta X}{\delta A_{a\mu}} \frac{\delta \Psi}{\delta \tilde{A}_{a\mu}^*} + \frac{\delta X}{\delta \tilde{A}_{a\mu}^*} \frac{\delta \Psi}{\delta A_{a\mu}} - \frac{\delta X}{\delta c_a} \frac{\delta \Psi}{\delta c_a^*} - \frac{\delta X}{\delta c_a^*} \frac{\delta \Psi}{\delta c_a} \right] \\ &\quad + \frac{\partial X}{\partial \alpha}.\end{aligned}\tag{29}$$

By Eq.(24) we see that at tree-level Ψ reduces to

$$\Psi = \int d^4x \frac{1}{2} \bar{c}_a b_a + O(\hbar).\tag{30}$$

In the present case Γ_0 in Eq.(22) is the vertex functional of Yang-Mills theory in the Landau gauge. Eq.(22) then allows one to express the coefficients of the α -expansion of 1-PI amplitudes in the Lorentz-covariant gauge in terms of 1-PI Landau gauge amplitudes plus an α -dependent contribution, arising from the generating functional Ψ .

The coefficient Γ_1 is obtained according to Eq.(22) by applying Δ_Ψ once on Γ_0 . Since Γ_0 does not depend on α , one obtains

$$\Gamma_1 = \int d^4x \left(\frac{\delta \Gamma_0}{\delta A_{a\mu}} \frac{\delta \Psi}{\delta \tilde{A}_{a\mu}^*} + \frac{\delta \Gamma_0}{\delta \tilde{A}_{a\mu}^*} \frac{\delta \Psi}{\delta A_{a\mu}} - \frac{\delta \Gamma_0}{\delta c_a} \frac{\delta \Psi}{\delta c_a^*} - \frac{\delta \Gamma_0}{\delta c_a^*} \frac{\delta \Psi}{\delta c_a} \right) \Big|_{\alpha=0}.\tag{31}$$

This equation expresses the linear approximation to Γ in powers of the gauge parameter α , in terms of amplitudes evaluated in the Landau gauge.

Let us now go on by computing Γ_2 , defined by

$$\Gamma_2 = \frac{\partial^2 \Gamma}{\partial \alpha^2} \Big|_{\alpha=0}.\tag{32}$$

According to Eq.(22), this is obtained by applying Δ_Ψ twice on Γ_0 and then setting $\alpha = 0$. Now we get two pieces: the first one again only contains amplitudes in the Landau gauge and can be written concisely as $\{\Psi, \{\Psi, \Gamma_0\}\}|_{\alpha=0}$. This is the term associated with naive exponentiation. However, there is also a contribution arising from the derivative of Ψ w.r.t. α , so that the full Γ_2 reads

$$\begin{aligned}\Gamma_2 &= \{\Psi, \{\Psi, \Gamma_0\}\}|_{\alpha=0} + \\ &\quad + \int d^4x \left[\frac{\delta \Gamma_0}{\delta A_{a\mu}} \frac{\delta^2 \Psi}{\partial \alpha \delta \tilde{A}_{a\mu}^*} + \frac{\delta \Gamma_0}{\delta \tilde{A}_{a\mu}^*} \frac{\delta^2 \Psi}{\partial \alpha \delta A_{a\mu}} \right. \\ &\quad \left. - \frac{\delta \Gamma_0}{\delta c_a} \frac{\delta^2 \Psi}{\partial \alpha \delta c_a^*} - \frac{\delta \Gamma_0}{\delta c_a^*} \frac{\delta^2 \Psi}{\partial \alpha \delta c_a} \right]_{\alpha=0}.\end{aligned}\tag{33}$$

5 Gauge Dependence of the Gluon Propagator

As an example, let us consider in the perturbative regime how one can derive the solution to the gauge evolution equation for the transverse part of the gluon propagator. For that purpose we introduce the transverse and longitudinal form factors according to

$$\Delta_{A_\mu^a A_\nu^b} = -i\delta^{ab} \left(\Delta_T(p^2) T^{\mu\nu} + \Delta_L(p^2) L^{\mu\nu} \right). \quad (34)$$

The relevant quantity is $\Delta_T(p^2)$. By taking two derivatives of Eq.(28) w.r.t. $A_{b_1\nu_1}, A_{b_2\nu_2}$ and then setting all fields and external sources to zero we obtain

$$\frac{\partial \Gamma_{A_{b_1\nu_1} A_{b_2\nu_2}}}{\partial \alpha} = - \int d^4x \left[\Gamma_{\theta \tilde{A}_{a\mu}^* A_{b_1\nu_1}} \Gamma_{A_{b_2\nu_2} A_{a\mu}} + \Gamma_{\theta \tilde{A}_{a\mu}^* A_{b_2\nu_2}} \Gamma_{A_{b_1\nu_1} A_{a\mu}} \right]. \quad (35)$$

In the above equation we have used the short-hand notation where lowstair letters denote functional differentiation w.r.t. that argument and it is understood that in the end one sets all fields Φ and external sources Φ^*, θ, θ_i equal to zero. For instance

$$\Gamma_{A_{b_1\nu_1} A_{b_2\nu_2}} \equiv \left. \frac{\delta^2 \Gamma}{\delta A_{b_1\nu_1} \delta A_{b_2\nu_2}} \right|_{\Phi=\Phi^*=\theta=\theta_i=0}. \quad (36)$$

Let us introduce transverse and longitudinal form factors for the 1-PI functions involved, namely (in the Fourier space)

$$\begin{aligned} \Gamma_{A_{b_1\nu_1} A_{b_2\nu_2}} &= \delta_{b_1 b_2} \left(G^T T_{\mu\nu} + G^L L_{\mu\nu} \right), \\ \Gamma_{\theta \tilde{A}_{a\mu}^* A_{b\nu}} &= \delta_{ab} \left(R^T T_{\mu\nu} + R^L L_{\mu\nu} \right). \end{aligned} \quad (37)$$

Then by applying the transverse projector to Eq.(35) one gets

$$\frac{\partial G^T}{\partial \alpha} = -2R^T G^T. \quad (38)$$

Let us denote by G_0^T the form factor in the Landau gauge. Then by integrating Eq.(38) one gets

$$G^T = \exp \left(- \int_0^\alpha 2R^T d\alpha' \right) G_0^T \quad (39)$$

and therefore for the transverse part of the gluon propagator

$$\Delta^T = \exp \left(\int_0^\alpha 2R^T d\alpha' \right) \Delta_0^T. \quad (40)$$

On the other hand, by Eq.(40) the following ratio

$$r = \exp \left(- \int_0^\alpha 2R^T d\alpha' \right) \frac{\Delta^T}{\Delta_0^T} \quad (41)$$

must be equal to one (and therefore gauge-independent).

While these results are valid in the perturbative expansion, their extension beyond perturbation theory is a subtle issue whose study is well beyond the scope of this work.

Several computations in the Landau gauge based on Schwinger-Dyson equations have indeed identified a scaling solution with $\Delta_T(0) = 0$ [31] and a decoupling one, with $\Delta_T(0) > 0$ (see Refs. [5, 6] and references therein).

It is therefore important to study what happens to these classes of solutions under a gauge variation, e.g. in order to compare the evolution with existing lattice results at $\alpha \neq 0$ [32].

If one were allowed to take the IR limit in both sides of the Eq.(40), $\Delta_0^T(0) = 0$ would imply that $\Delta^T(0)$ is also equal to zero. I.e. if a solution to the QCD Schwinger-Dyson equations is of the scaling type in the Landau gauge, it would also be scaling in a Lorentz-covariant gauge. Moreover, for massive solutions the sign of $\Delta^T(0)$ would be gauge-independent, as a consequence of Eq.(40).

However the validity of Eq.(40) beyond perturbation theory is questionable. In particular, the presence of IR divergences in the explicit non-perturbative evaluation of the form factor R^T might destroy the validity of the assumption that the amplitudes are analytic around the Landau gauge point $\alpha = 0$. In this case the Lie series solution in Eq.(22) cannot be any more used to reconstruct the vertex functional in a gauge $\alpha \neq 0$.

6 Conclusions

The existence of a canonical flow in the space of gauge parameters and the related solution in terms of a Lie series provide a way to compare results in different gauges within an algebraic framework that is bound to hold even beyond perturbation theory (as far as the ST identity is valid).

The dependence of the generating functional of the canonical flow on the gauge parameter prevents to get the full solution by a naive exponentiation. Such a solution can be expressed through an appropriate Lie series, in close analogy to the solution of the extended ST identity in the presence of a background gauge connection.

Knowing such a Lie series eases the comparison between computations carried out in different gauges. In the simplest example of the 2-point gluon function, a closed formula interpolating between the Landau and the Lorentz-covariant gauge can be obtained, under the assumption that analyticity in the gauge parameter around $\alpha = 0$ holds.

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